Supervisory control for optimal route guidance in intelligent vehicle highway systems based on hybrid network models

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This paper is devoted to the problem of optimal route guidance in intelligent vehicle highway systems to provide travellers with information minimizing the travel time as a kind of advanced traveller information systems. In order to deal with this problem, we first introduce a hybrid network model by incorporating the high-level discrete event model with the low-level flow dynamics model. Based on it, two feasible control policies are proposed employing supervisory control theory and then followed by a comparative simulation study.

1. Introduction

Intelligent vehicle highway systems (IVHSs) were introduced to address effectively the needs of a roadway complex through the integration of many emerging technologies including information processing, communications and control (Fenton 1994, Hedrick et al. 1994). IVHSs are composed of several interrelated areas: advanced traveller information systems (ATISs), advanced vehicle control systems (AVCSs), automated highway systems (AHSs), advanced traffic management systems (ATMSs), etc. Among these, the AHS–ATMS concept is receiving considerable attention as a means of alleviating some of the highway problems and resulting in improvements in traffic throughput and decreases in congestion. In particular, numerous studies of highway network traffic control or vehicle management have been made up to the present (Benmohamed and Meerkov 1994, Fenton 1994, Hedrick et al. 1994, Alvarez et al. 1996, Febbraro and Sacone 1997, Iftar 1997, Porche and Lafortune 1998); however, no preferred approach to route guidance based on systematic routing schemes under varying traffic situations has been proposed. In this paper, we study the problem of optimal route guidance in a metropolitan highway network in view of providing travellers with better information, resulting in reduced travel time and traffic congestion. This could be interpreted as a means for steering traffic towards user optima that utilizes alternate feasible routes regarding ATISs. Throughout the developments of this paper, it is assumed that the guided vehicle moves following the route provided by a supervisor and that the movements of all the other vehicles are dynamic and instantaneous, that is they select routes dynamically at each intersection with the objective of minimizing instantaneous travel time to the next intersection (Hall 1996). Deriving the optimal route in the dynamic network requires elusive information concerning predictions of all vehicle behaviours and becomes nondeterministic polynomial (NP) complete. To deal with this problem, we first propose a hybrid network model of the metropolitan highway network consisting of the high-level weighted discrete event model (Heymann 1990) and the low-level network flow dynamics model and then develop two supervisory control policies based on this model ensuring suboptimal routes of a shade of difference with polynomial order computational complexities as another approach to the optimal route guidance problem. The performances of the proposed control policies are compared with those of optimal routing and some conventional routing schemes through a simulation study.

2. Weighted discrete event models and supervisory control

In the supervisory control framework for discrete event dynamic systems (Ramadge and Wonham 1987,
Wonham and Ramadge 1987), the plant to be controlled is modelled by an automaton $G = (\Sigma, Q, \delta, q_0, Q_m)$ where $\Sigma$ is an alphabet of event labels, $Q$ is a set of states, $q_0 \in Q$ is the initial state, $Q_m \in Q$ is the set of marker states and $\delta: \Sigma \times Q \rightarrow Q$, the transition function, is a partial function defined at each state in $Q$ for a subset of $\Sigma$. The behaviour of $G$ is characterized by a set of sequences of events, $L(G)$, called a language. In this paper, we introduce a weighted automaton model of the highway network $N$, $G_N = (G, W)$, to consider the roadway traffic as a weight parameter $W$ of the automaton model, in which $W: \Sigma \times Q \rightarrow \mathbb{R}^+$ where $\mathbb{R}^+$ is the positive integer including 0 and $\mathbb{R}^+$ is the positive real including 0. A supervisor is then an agent which observes a sequence of events as it generated by $G$ and enables or disables any of the controllable events at any point in time throughout its observation. By performing such a manipulation of controllable events, the supervisor ensures that only a subset $K$ of $L(G)$ is permitted to be generated. Formally, a supervisor, $S_K$ achieving $K$ is a pair $(S_K, \phi)$ where $S_K$ is an automaton which recognizes a legal language $K$ over the same event set as the plant $G_N$ and $\phi$, called a feedback map, is a map from the event set and states of $S_K$ to the set $\{1(\text{enable}), 0(\text{disable})\}$. If $X$ denotes the set of states of $S_K$, then $\phi: \Sigma \times X \rightarrow \{1, 0\}$. The automaton $S_K$ tracks and controls the behaviour of $G_N$. It changes its states according to the events generated by $G_N$ and in turn, at each state $x$ of $S_K$, the control rule $\phi(\sigma, x)$ dictates whether $\sigma$ is to be enabled (i.e., $\sigma \in \Sigma_{\text{enable}}$) or disabled ($\sigma \in \Sigma_{\text{disable}}$) at the corresponding state of $G_N$ to achieve $K$.

3. Optimal route guidance problem in intelligent vehicle highway systems

Consider the highway network modelled by a weighted automaton $G_N$ in figure 1. Note that any other configuration of the network can be transformed into the equivalent regular square topologies in figure 1. In this case, $Q = \{q_{i,j}\mid i \in [1, i_m], j \in [1, j_m]\}$ and $\sigma \in \Sigma = \{e_t, e_u\}$. Hereafter, we use the following notation: $W_c$ for the capacity of the road $(W(\cdot, \cdot) \leq W_c)$, $W_s$ for maximum weight ensuring maximum vehicle speed, $W_e$ for $W(q, \sigma_e, |s|)$ where $q$ stands for $q_{i,j}$ for some $i$ and $j$, $\sigma_e \in \Sigma_{\text{enable}}$, and $|s|$ implies the cardinality of the string $s$ (the number of events comprising the string $s$), $W^i_j$ for $W(q_{i,j-1}, e_t, \cdot)$, $W^{e_u}$ for $W(q_{i,j-1}, e_u, \cdot)$, $W^{e_t}$ for $W(q_{i,j-1}, e_t, \cdot)$, $W^{e_u}$ for $W(q_{i,j-1}, e_u, \cdot)$, $W_u$ for the number of vehicles going out of the road during one update period, $\Delta W_{i,j}^{\text{out}} (k = 1, 2)$ for the number of vehicles going out of the road having $W_{i,j}^u$ during one update period, $\Delta W_{i,j}^{\text{in}} (k = 1, 2)$ for the number of vehicles coming into the road having $W_{i,j}^u$ during one update period, and $W_r(k)$ for the current weight of the road under control (e.g. assume $\sigma_e$ with $\delta(\sigma_e, q_{i-1}) = q_{i,j}$) after the $k$th update (i.e. for $\sigma_e$, $W_r(k) = W_e - \sum_{i=1}^{\Delta} W_{i,j}^{\text{out}}$ with $W_r(0) = W_e$ and $W_r(n) = 0$).

3.1. Assumptions

Throughout this paper, we assume the following for the network flow dynamics and the concomitant weight changes.

(A 1) Vehicles in one segment of the roads are distributed with even spacing and the same speeds.

(A 2) Each road is one way; therefore only unidirectional transition is allowed.

(A 3) The transportation delay at the intersection is small enough to be neglected.

(A 4) The number of vehicles crossing the intersection is preserved, that is $\Delta W_{i,j}^{\text{out}} + \Delta W_{i,j}^{\text{in}} = \Delta W_{i,j}^{\text{out}} + \Delta W_{i,j}^{\text{in}}$.

(A 5) In the boundary of the network, $W_{i,j}^u = W_{i,j}^{\text{in}} = (W_{i,j}^u + W_{i,j}^{\text{in}})/2$ for $i = i_m$ or $j = j_m$.

3.2. Network flow dynamics

The number of vehicles going out of the road during one update period, $W_u = W_s$ if $W_r(k) \geq W_s$, and

![Figure 1](image-url)  
Figure 1. The weighted automaton model of the highway network.
the total number of vehicles coming into the road having \(W_{ij}^{o1}\) during the transition from \(|s| - 1\) to \(|s|\) (composed of \(n\) update periods). Then the weight changes of each road are as follows.

\[
W_{ij}^t(\cdot, \cdot, |s|) = W_{ij}^t(\cdot, \cdot, |s| - 1) + \Delta W_{ij}^{o1}(\cdot, \cdot, |s| - 1) - \Delta W_{ij}^{o2}(\cdot, \cdot, |s| - 1),
\]

This rule of change means that the number of vehicles going out of the road having \(W_{ij}^k\) during one update period is as many as the current weight of the road if the capacity of the outgoing roads at the intersection is enough, and the equally allocated amount of the available capacity otherwise.

3.3. Optimal route guidance problem

Under the foregoing assumptions (A1)–(A5), the optimal route guidance problem becomes designing a supervisor \(S_{K_{opt}}\) such that the closed loop system \(S_{K_{opt}}/G_N\) results in the optimal route \(K_{opt}\) from the source to destination in view of weight sum minimization. Let \(J(L_K) = \sum_{s \in T_K} f(W(\cdot, \cdot, |s|))\) where \(L_K = L_m(S_K/G_N)\) and \(f(W) = (n - 1)W_s + W_e(n - 1)\) with \(W_i(0) = W_i, W_i(k) = W_i(k - 1) - \Delta W_{s_{out}}\), and \(W_i(n) = 0\). Formally, the optimal route guidance problem can be stated as finding \(S_K\) such that \(\min_K [J(L_K)]\). Then, as assumed in the outset, the guided vehicle is to move along the route provided by the supervisor.

4. Control policies

The previous sections have been devoted to formulating the high-level weighted discrete event model and the low-level network flow dynamics model of the highway network, resulting in a hybrid network model. In this section, we propose the following control policies to the optimal route guidance problem based on this hybrid network model and supervisory control theory.
Together with the foregoing network flow dynamics, we observe that each weight of the road edge inclines to converge to the average over the branches of the intersection from the initial weight traffic. We let the weighted automaton having these steady-state average weights be an average weighted automaton $\overline{G}_N$. If we compute the minimum-weight-sum route (to be called the rough path) over this $\overline{G}_N$, then there will be a discrepancy with the optimal route until the weights converge. Therefore, we recompute the minimum-weight-sum route over the simulated hybrid network model following the rough path and synthesize the supervisor based on this route. Accordingly, we formulate the following control policy 1.

**Control Policy 1 (CP1):**

$$S_{K_{CP1}} = (S, \phi) \text{ with } \phi(\sigma, x) = \begin{cases} 1, & \text{if } l\sigma \in \Gamma_{CP1}, \\ 0, & \text{otherwise}, \end{cases}$$

for each $\sigma \in \{e_1, e_n\}$, $x = q_{ij}$ with $i \in [1, i_m]$ and $j \in [1, j_m]$, and $l \in \Gamma(S_{K_{CP1}} / G_N)$ where $K_{CP1} = K$, if

$$\sigma_i (K_{CP1}) = \sigma_j (K)$$

($\sigma_i (s)$ is the $n$th event comprising the string $s$) and $K_{CP1} = K_{m}$ else if

$$\sigma_1 (K_{CP1}) = \sigma_1 (K_m)$$

for $K = \arg \min_j [J'(L(G_N))]$ with $J'(l_K) = \sum j \epsilon T_k W(e, s_j)$ and $\overline{G}_N = (G, \overline{W})$ where $\overline{W}$ means $W_{ij}^0 = W_{ij} = (\sum_k = 1 W_{ij}^k)/2$ for each $q_{ij}$, $K_{m} = \arg \min_j [J'(L(S_K / G_N))]$, and $K_{CP1} = \arg \min_j [J'(L(S_K / G_N))]$.

Instead of recomputing the minimum weight sum route on the simulated hybrid network model along the rough path, the discrepancy with the optimal route can be compensated by repeating the computation over $\overline{G}_N$ at each transition step with the newly updated weight traffic. In this respect, we propose the following control policy 2.

**Control Policy 2 (CP2):**

$$S_{K_{CP2}} = (S, \phi) \text{ with } \phi(\sigma, x) = \begin{cases} 1, & \text{if } l\sigma \in \Gamma_{CP2}, \\ 0, & \text{otherwise}, \end{cases}$$

for each $\sigma \in \{e_1, e_n\}$, $x = q_{ij}$ with $i \in [1, i_m]$ and $j \in [1, j_m]$, and $l \in \Gamma(S_{K_{CP2}} / G_N)$ where $K_{CP2} = e_1 e_2 \cdots e_n$ with $e_i = \sigma_1 (K_{CP1})$ and $e_p = \sigma_p (\arg \min_j [J'(L(S_{e_1 e_2 \cdots e_{p-1} / G_N})])$ for $p \in [2, n]$, $(n = |K_{CP2}|)$.

CP1 navigates along the minimum weight sum path route on the simulated network flow dynamics through the virtual minimum-weight-sum path of $\overline{G}_N$, computed off line. On the other hand, CP2 makes the decision at each transition step dynamically based on renewed average information of $\overline{G}_N$. For CP1, the upper bound of error $E_{CP1}$ between $J(S_{K_{CP1}})$ and $J_{s}(S_{K_{CP1}})$ is $\sum_{e \in E} \sum_{j \in T_k} (J(\overline{W}) - W) + \varepsilon$ where $\varepsilon$ is a round-off error around 1% of $W$. In case of CP2, $E_{CP2} \leq \sum_{e \in E_k} |W - \overline{W}|$.

**Remark 1—Computational complexity issues:** In order to compare the computational complexity of the proposed control policies with the conventional routeing schemes, we consider optimal routeing (OR), static routeing (SR) and simple control law (SCL). Here, OR means obtaining the route resulting in the minimum weight sum (which is not precomputable with polynomial order computational complexity), SR means deriving the route that seems to be optimal at the outset, and SCL the decision scheme repeating SR at each transition just regarding the current state as new source node. Assume that $|Q| = n^2$; then the computational complexity of CP1 and SR is $O(n^3)$ and CP2 shows $O(n^2)$ as well as SCL when employing the Dijkstra algorithm (Hu 1982) for an internal step of finding the shortest path between fixed weighted nodes. For OR, it becomes $O((2n)!/2(n!)^2) \geq O(2^n)$; hence the problem is NP-complete.

5. Simulation study

In the following, we compare the performances of the proposed control policies with those of OR, SR and SCL through a simulation study. The comparison of the averaged weight sum (average over 100 times simulation with randomly selected initial traffic weights) for each scheme is shown in figure 2 with $|Q| = 25, i_m = 5, j_m = 5, W_c = 85$ and $W_s = 15$. An enlargement of figure 2 at each transition step is shown in figure 3. From figures 2 and 3, we know that CP1 and CP2 show almost the same performance as OR within 0.1% error bounds. Furthermore, the comparison of the performance in the case of an abnormal situation such as the occurrence of accidents is illustrated in figures 4 and 5. CP2 reveals a better performance than CP1 does since CP2 embodies a dynamic routeing scheme which is capable of reconfiguring the route according to the varying traffic situation. Note that in this simulation study we focused on the performance measure of the guided vehicle by the supervisor and considered all the other dynamic and instantaneous types of vehicles as moving according to the previous network flow dynamics.

6. Conclusions

In this paper, the optimal route guidance problem in IVHS has been investigated on the basis of hybrid network models and supervisory control theory. Two control policies (CP1 and CP2) with polynomial order computational complexities have been proposed and their performances have been compared with those of the simple static and dynamic routeing-based methods and the optimal routeing scheme through the comparative simulation study. In most cases, CP1 and CP2 have shown the optimal route guidance results and, in par-
Figure 2. Comparison of the averaged weight sum of each scheme.

Figure 3. Enlargement of the compared results in figure 2.
Figure 4. Typical comparison in a normal traffic situation.

Figure 5. Comparison in an abnormal traffic situation.
ticular for CP2, the average dynamic routeing scheme has revealed better performance in the case of unexpected network congestion such as the occurrence of accidents.

References


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