Self-tuning control of electromagnetic levitation systems


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Abstract

The mass uncertainty resulting from taking passengers, loading freight, and being disturbed by external forces is the main reason why the performance of electromagnetic levitation systems deteriorate. Thus, this paper proposes a self-tuning controller for electromagnetic levitation systems with unknown mass variations. The self-tuning controller is an extended version of gain scheduling for the general case of unknown scheduling parameters. Experimental results show the benefits of the scheme in cases of unknown mass variation compared with a conventional control methodology.

Keywords: Electromagnetic levitation systems; Self-tuning controller; Mass estimator; Scheduling parameter; Conventional state feedback controller

1. Introduction

Nowadays, magnetically levitated transport systems are receiving increasing attention around the world. Electromagnetic levitation systems, which are highly nonlinear and have unstable dynamics have been researched because the absence of contact reduces noise, component wear, vibration, maintenance costs, etc. Mass uncertainty and external force disturbance are the main reasons for the deterioration of the quality of system performance. In order to solve such problems, classical state-feedback control with pole-placement has been applied to the linearized models corresponding to specific operating conditions (Sinha, 1987). However, since the operating conditions change according to the mass variation and the force disturbance, its local controller could not achieve satisfactory performance with global operating points. Recently, nonlinear control schemes such as gain scheduling (Kim & Kim, 1994) and feedback-linearization methodology (Joo & Seo, 1997; Trumper, Olson & Subrahmanyan, 1997) have been reported in the literature. However, the force disturbance estimator (Kim & Kim, 1994) required the precise inverse dynamics of the given plant model using the derivative information of the state vectors. Thus, it was confined to solving regulation problems of only force disturbances with an assumption of no mass variation. Moreover, though the feedback linearization canceled the nonlinear nature of the electromagnetic levitation system, it was limited when tackling the unknown mass variation (Joo & Seo, 1997; Trumper et al., 1997). Thus, this paper proposes a self-tuning controller which is an extended version of gain scheduling. It is applied to a levitation system with mass uncertainty in order to guarantee its robust performance against unknown mass variations.

The gain-scheduled controller (Huang & Rugh, 1990; Rugh, 1991; Huang & Rugh, 1992a; Kaminer, Pascoal, Khargonekar & Coleman, 1995) was developed and proved to be a successful design methodology in many engineering applications. Such results had a common limitation in which the resulting controllers were valid only for constant or sufficiently slow varying parameters. More recently, to improve its regulation performance for fast varying parameters, Sureshbabu and Rugh (1995), and Lee and Lim (1997) have proposed extended control laws using the derivative information of the time-varying parameter. The concept of a kth-order approximate equilibrium point was introduced and its kth-order robust control law was constructed in uncertain nonlinear systems with time-varying parameters (Huang & Rugh, 1992b; Huang, 1995). Nowadays, $H_\infty$ control theory has become an effective design methodology in tracking the problem of stability and performance robustness under plant uncertainties. Many researchers have been interested in the fusion technique of $H_\infty$ synthesis and gain...
scheduling (Nichols, Reichert & Rugh, 1993; Apkarian & Gahinet 1995; Lu & Balas, 1995; Helmersson, 1996). However, note that the stabilizing control gains of all the gain scheduled controllers mentioned above are driven by the measured scheduling parameter. Thus, cases where it is difficult to measure the parameter at every control sequence, these controllers become useless and the unknown parameter has to be estimated and engaged appropriately.

The proposed self-tuning controller in this paper consists of the mass estimator and the scheduled controller. The mass estimator asymptotically tracks the unknown mass (scheduling parameter) variation in the closed-loop control system, and the gains of the stabilizing controller are appropriately scheduled according to the estimated mass. The proposed estimator has several advantages when compared to those in the related literature. Different from other estimation methods (e.g. Hsia, Lasky & Guo, 1991; Chang & Lee, 1996), it does not require the state vector time derivative that induces noise in many applications. Second, it does not introduce any auxiliary system that estimates the perturbed plant-state vector or the unknown system parameter (e.g. Matsui & Makino, 1993; Lu & Chen, 1995). Third, it has a simple structure that minimizes the burden of computation and is applicable to the existing gain scheduled controllers. In the following section, the open-loop dynamics of a single-axis electromagnetic levitation system is presented. Section 3 deals with the construction of the self-tuning controller and mass estimator in the considered levitation system. In Section 4, the experimental setup is described and a comparison of the experimental results of the proposed controller and a classical state feedback controller is made. The conclusion is summarized in Section 5.

2. Model of electromagnetic levitation systems

The technology of magnetically levitated transport systems is divided into the mechanical technology of cabin, bogies and secondary suspension system, the electrical technology of the levitation and propulsion system, and the construction technology for guideway. The levitation system that this paper focuses on is composed of a group of controlled levitation magnets. The primary causes of its performance deterioration are the mass uncertainty and the external force disturbance. The mass variation results from taking passengers, loading freight, or the failure of any one of the controlled levitation magnet groups. Furthermore, there is force disturbance from the wind force, the lateral force, or the centrifugal force around corners. If the force disturbance is assumed to lead to an equivalent mass variation, its solution is inducible through the mass uncertainty problem in many cases.

The single-axis model of the electromagnetic levitation system in Fig. 1 is described by the following nonlinear dynamic equation (Sinha, 1987):

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= -\frac{\mu_0 N^2 A}{4m} \left(\frac{x_3}{x_1}\right)^2 + g, \\
\dot{x}_3 &= \frac{x_2 x_3}{x_1} - \frac{2R}{\mu_0 N^2 A} x_1 x_3 + \frac{2x_1}{\mu_0 N^2 A} u, \\
y &= x_1,
\end{align*}
\]

where \(x_1\) is the vertical air gap, \(x_2\) is the vertical velocity, \(x_3 = i(t)\) is the magnet current, \(u = v(t)\) is the applied voltage, \(m\) is the total mass (nominally \(m = m_0 = 300\) kg), \(N = 660\) is the number of turns of the coil wrapped around the magnet, \(A = 0.04\) m² is the pole area, \(\mu_0 = 4\pi \times 10^{-7}\) H/m is the permeability of free space, \(R = 1\) Ω is the coil resistance, \(g = 9.8\) m/s² is the gravity constant, and \(r_d\) is the reference air gap.

For convenience of notation, (1) is rewritten as

\[
\dot{x} = f(x, u, m),
\]

\[
y = h(x).
\]

3. Designing the self-tuning controller

3.1. Construction of the scheduled controller

Defining the unknown mass as the scheduling parameter, the smooth function pair \((x(m), u(m))\) satisfying

\[
0 = f(x(m), u(m), m) \quad \text{and} \quad r_d = h(x(m))
\]

for \(m \in \Gamma = \{m \in R \mid m > 0\}\) is computed by

\[
x(m) = \begin{bmatrix} r_d \\ 0 \\ \frac{2r_d}{N} \frac{gm}{\sqrt{\mu_0 A}} \end{bmatrix} \quad \text{and} \quad u(m) = \frac{2r_d R}{N} \sqrt{\frac{gm}{\mu_0 A}}.
\]

Note that the nominal value of the magnet current (voltage) is computed by \(7.5A\) (V) for \(r_d = 10\) mm and \(m = m_0\). The control objective is to obtain \(\lim_{t \to \infty} ||r_d - y|| = 0\) while rejecting the unknown mass \(m\).
Then, for each fixed $m$, the corresponding linearized closed-loop system (2) with a nonlinear state feedback control law $u = k(x, m)$ is written in the form of

$$\dot{x}_d = A(m)x_d + B(m)u_d,$$
$$y_d = C(m)x_d,$$
$$u_d = K(m)x_d,$$

where the deviation variables are given by $x_d = x - x(m)$, $u_d = u - u(m)$, and $y_d = y - r_d$. The linearized system coefficients are given by

$$A(m) = \frac{\partial f(x(m), u(m), m)}{\partial x} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2g}{r_d} & 0 & -\frac{N}{r_d} \sqrt{\frac{Ag\mu_0}{m}} \\ 0 & 2 \frac{gm}{N} \sqrt{\frac{2}{A\mu_0}} & -\frac{2r_dR}{A\mu_0 N^2} \end{bmatrix},$$

$$B(m) = \frac{\partial f(x(m), u(m), m)}{\partial u} = \begin{bmatrix} 0 & 0 & \frac{2r_d}{A\mu_0 N^2} \end{bmatrix},$$

$$C(m) = \frac{\partial f(x(m))}{\partial x} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

and the linearized control law coefficients by $K(m) = \frac{\partial k(x(m), m)}{\partial x}$. In order to achieve the desired pole-placement in the linearized closed-loop system, $K(m)$ is determined so that the eigenvalues of $A(m) + B(m)K(m)$ have specified values with negative real parts for each $m \in \Gamma$ with the controllable pair of $A(m_0)$ and $B(m_0)$ (Rugh, 1991). Then

$$k(x, m) = u(m) + K(m)(x - x(m)) \tag{5}$$

is obtained with

$$K(m) = \begin{bmatrix} -\lambda N(6g + \lambda^2 r_d) & \frac{\mu_0 Am}{g} \frac{3\lambda^2 N}{2} & \frac{\mu_0 Am}{g} \\ 2r_d & 2r_d & \frac{3A\lambda\mu_0 N^2}{2r_d} + R \end{bmatrix},$$

where $\lambda = -6.5$ is the triple eigenvalues of the linearized closed-loop system. As shown in Fig. 2, the unknown mass $m$ in (5) is substituted with the estimated mass in the closed-loop control system. For comparison, the conventional state feedback controller is introduced (Sinha, 1987) so that

$$u = \frac{2r_d R}{N} \sqrt{\frac{gm_0}{\mu_0 A}} + k_1(x_1 - r_d) + k_2 x_2$$

$$+ k_3 \left( x_3 - \frac{2r_d}{N} \sqrt{\frac{gm_0}{\mu_0 A}} \right), \tag{6}$$

where the feedback gains $k_1$, $k_2$, and $k_3$ are determined so that the eigenvalues of the linearized closed-loop system with (6) at $m = m_0$ are identical to those with (5).

### 3.2. Construction of the mass estimator

It is noted that since the unknown mass $m$ drives the controller $k(x, m)$, it should be appropriately estimated and engaged. Using the idea that the control objective is achieved only by asymptotic tracking, i.e., $\lim_{t \to \infty} ||x - x(m)|| = \lim_{t \to \infty} ||u - u(m)|| = 0$ for $m \in \Gamma$ in (5), an error function such as

$$e(t) = \int_0^t F(x(\tau), u(\tau), \hat{m}(\tau)) d\tau \tag{7}$$

is defined, with

$$F(x, u, \hat{m}) = \sum_{i=1}^3 (x - x(\hat{m}))_i + (u - u(\hat{m}))$$

$$= (x_1 - r_d) + x_2 + \left( x_3 - \frac{2r_d}{N} \sqrt{\frac{gm}{\mu_0 A}} \right)$$

$$+ \left( u - \frac{2r_d R}{N} \sqrt{\frac{gm}{\mu_0 A}} \right), \tag{8}$$

where $\hat{m}$ is the estimated value of unknown $m$.

Define $\Delta \hat{m}$ as an estimated value of $\Delta M = \sqrt{m} - \sqrt{m_0}$. Letting $\sqrt{\hat{m}} = \sqrt{m_0} + \Delta \hat{m}$ and using...
a Taylor series expansion of (7) at \( \sqrt{m} = \sqrt{m_0} \),
\[
\dot{e} = \sum_{i=1}^{3} (x_i - x_i(\hat{m})) + (u - u(\hat{m}))
\]
\[
= \sum_{i=1}^{3} (x_i(m_0)) + (u(m_0))
\]
\[
- \left( \sum_{i=1}^{3} \left( \frac{\dot{x}_i(m_0)}{\dot{\sqrt{m}}} + \frac{\dot{u}(m_0)}{\sqrt{m}} \right) \right) \Delta \dot{M} + o(\Delta \dot{M})
\]
\[
= F(x, u, m_0) - \left( \frac{2r_d(1 + R)}{N} \sqrt{\frac{g}{\mu_0 A}} \right) \Delta \dot{M} + o(\Delta \dot{M}). \tag{9}
\]
In order to construct the mass estimator, an estimation law of \( \dot{M} \) is proposed so that
\[
\Delta \dot{M} = \Psi(e) = \frac{x}{(2r_d(1 + R)/N)\sqrt{g/\mu_0 A}} e \tag{10}
\]
with \( x > 0 \).
Letting \( \dot{\Delta} \dot{M} = \dot{M} \) in (10),
\[
e_{sa} = \Psi^{-1}(\Delta \dot{M}) = \frac{\Delta M}{2r_d(1 + R) N \sqrt{g/\mu_0 A}} \tag{11}
\]
To ensure that \( \lim_{t \to \infty} e(t) = e_{sa} \), the Lyapunov function candidate \( V = 0.5(e - e_{sa})^2 \) is differentiated with respect to time. Then, from (9) and (10),
\[
\dot{V} = (e - e_{sa})\dot{e} = -(e - e_{sa})(x - \zeta(e)) \tag{12}
\]
is obtained, where \( \zeta(e) = F(x, u, m_0) + o(\Delta \dot{M}) \).
From the fact that \( x e_{sa} - \zeta(e_{sa}) = 0 \), it follows that for \( x > 0 \),
\[
x e - \zeta(e) > 0 \quad \text{if} \quad e > e_{sa},
\]
\[
x e - \zeta(e) < 0 \quad \text{if} \quad e < e_{sa} \tag{13}
\]
as shown in Fig. 3. Note that condition (13) is satisfied regardless of the sign of \( e_{sa} \). Thus, using (13) it is confirmed that \( \dot{V} < 0 \) is satisfied for \( e \neq e_{sa} \) in (12) and the error function with the estimation law (10) converges to \( e_{sa} \).

Let \( \tilde{x}_d = x - x(\hat{m}) \) and \( \tilde{u}_d = u - u(\hat{m}) \). Since \( \lim_{t \to \infty} e(t) = e_{sa} \) and \( \lim_{t \to \infty} \dot{e}(t) = 0 \), from (8)
\[
F(x, u, \hat{m}) = K_1(\hat{m})\tilde{x}_d + K_2(\hat{m})\tilde{u}_d + K_3(\hat{m})\tilde{x}_d = 0 \tag{14}
\]
with \( K_1(\hat{m}) = 1 + K_1(\hat{m}) \). Then, using \( \tilde{x}_d \), in (14) the reduced system of the linearized system (4) is rewritten as
\[
\dot{\tilde{x}}_d = x_d,
\]
\[
\dot{\tilde{x}}_d = \frac{2g}{r_d} \tilde{x}_d - \frac{N}{r_d} \sqrt{\frac{Ag\mu_0}{m}} \tilde{x}_d + \frac{N}{r_d K_3(m)} \sqrt{\frac{Ag\mu_0}{m}} w \tag{15}
\]
with
\[
x(m) = \frac{2g}{r_d} + \frac{NK_1(m)}{r_d K_3(m)} \sqrt{\frac{Ag\mu_0}{m}},
\]
\[
\beta(m) = \frac{NK_2(m)}{r_d K_3(m)} \sqrt{\frac{Ag\mu_0}{m}},
\]
\[
w = \sum_{i=1}^{3} (K_1(\hat{m})\tilde{x}_d - K_1(\hat{m})\tilde{x}_d)
\]
and it is confirmed that \( x(m) < 0 \) and \( \beta(m) < 0 \) for \( m \geq m_0 \). Furthermore, from the fact that
\[
\lim_{t \to \infty} e(t) - e_{sa} = \lim_{t \to \infty} \int_{0}^{t} (F(x, u, \hat{m}) - F(x, u, m)) d\tau
\]
\[
= \lim_{t \to \infty} \int_{0}^{t} w(\tau) d\tau = 0
\]
it can be concluded that \( \lim_{t \to \infty} w(t) = 0 \). Thus, the reduced system (15) has a stable equilibrium point at \( x_1 = x_1(m) \) and \( x_2 = x_2(m) \) against the vanishing signal \( w \) and it follows that \( \lim_{t \to \infty} (x - x(m), u - u(m)) = (0, 0) \). Finally, since the asymptotic tracking is achieved only for \( (x(\hat{m}), u(\hat{m})) = (x(m), u(m)) \) and the smooth function pair \( (x(\cdot), u(\cdot)) \) is invertible for \( m \in \Gamma \), \( \hat{m} \) is uniquely determined to be \( m \).

**Remark 1.** Note that if the magnet current sensor value, \( x_3 \), is not available due to the sensor failure, noise, etc., it can be calculated by
\[
x_3 = \frac{2r_d}{N} \sqrt{\frac{g\hat{m}}{\mu_0 A}} + \frac{2}{N} \sqrt{\frac{g\hat{m}}{\mu_0 A}} (x_1 - r_d) - \frac{r_d}{N} \sqrt{\frac{\hat{m}}{\mu_0 Ag}} \tilde{x}_2, \tag{16}
\]
where \( \tilde{x}_2 \) is directly measured from the accelerometer. From (8) and (16), \( F(x, u, \hat{m}) = (x_1 - r_d) + x_2 + \tilde{x}_2 + (u - (2r_d R/N) \sqrt{g\hat{m}/\mu_0 A}) \) is obtained. The relation of (16) is derived by linearizing the second equation of (1) about \( x_1 = r_d \) and \( \tilde{x}_2 = 0 \).
4. Experimental validation

4.1. Experimental setup

The experimental setup is shown in Figs. 4 and 5 for the verification of the proposed control algorithm. To enable experimental evaluation of the self-tuning control, a digital control system based on Power PC (PPCIA 604e, 225 MHz) was constructed to enable high-speed sampling. The vertical air gap is measured by an inductive-type gap sensor with an effective range of 0–25 mm and the acceleration of the magnet is measured by a servo accelerometer with an effective range of 0 – ± 3 g. Furthermore, a Hall-type sensor is used for measuring the magnet current. The power source is a 10 kHz PWM voltage-type chopper with a dc link voltage of 300 V. A Pentium with a Power PC (PPCIA 604e) board using VxWorks Tornado performs the control algorithm, as well as data acquisition and signal conditioning. Using this computer, both the proposed self-tuning controller and the conventional state feedback controller are implemented at a sampling rate of 2 kHz.

The acquisition of accurate physical values is essential to the validation of the proposed control scheme. Due to environments with high levels of electromagnetic interference, signal-conditioning filters are required. Using a spectrum analyzer, the frequency band of the measurement noise of EMS system was measured to be between 800 Hz and 5 kHz. Hence, in order to filter out the measurement noise, a low-pass analog filter that has an appropriate set of specifications was designed. The vertical gap, acceleration, and magnet current measured by the voltage unit, are conditioned by low-pass analog filter and then are sampled by 12-bit analog to digital (A/D) converters. Particularly, the vertical acceleration needs to calibrate the dc offset in the accelerometer, which is for gravity. Since the velocity sensor is not available, the velocity of magnet is converted from the integrated acceleration value with high-speed sampling through the PPCIA 604e. The offset error in the accelerometer varies

![Fig. 4. Block diagram of the overall control system.](image)

![Fig. 5. Photograph of the experimental hardware.](image)

![Fig. 6. Regulation control for \( r_g = 10 \text{ mm} \) without any additional mass. (a) Using conventional state feedback controller. (b) Using self-tuning controller.](image)
according to the voltage of PWM chopper, though this may be minimized by the preceding calibration. Thus, an integrator in the form of \( s/(s^2 + 1.30s + 8.87) \) is used to prevent saturation of the integrator as a result of the offset error in the accelerometer.

4.2. Experimental results

Both control schemes were experimentally investigated on the electromagnetic levitation system with step variations of mass. The transient responses for the reference air gap \( r_d \) from 19 to 10 mm are shown in Fig. 6 using the conventional state-feedback controller and the proposed controller with \( \alpha = 2 \times 10^5 \). The large \( \alpha \) results in fast estimation but large oscillatory behavior, whereas a small value results in slow estimation but small oscillatory behavior. Thus, the value of \( \alpha \) should be selected to navigate the speed-oscillatory trade-off. It is observed that the transient behaviors of Figs. 6(a) and (b) are almost identical when no additional mass is added. Fig. 7 shows output responses when an additional mass of 300 kg is introduced at \( t = 2 \) s. It can be clearly observed that the proposed control scheme shows an improved output performance with small high-frequency resonance in comparison with the conventional one. Moreover, the estimated mass that asymptotically tracks the unknown mass is plotted in Fig. 8. Both controllers were also implemented to track the reference air-gap \( r_d = 10 \) mm + 2.5 \( \sin(2\pi t) \) mm and their output responses are shown in Fig. 9. In Fig. 10, the output responses with an additional mass of 300 kg are plotted. It is also noted that the proposed control scheme shows better reference-tracking performance than the conventional one.

5. Conclusions

A self-tuning electromagnetic levitation system is developed and implemented to improve its performance against unknown mass variations. The existing gain-scheduled control systems are driven and controlled by the measured scheduling parameter. However, the proposed self-tuning controller, consisting of a mass estimator and a scheduled controller, is an extended form of existing gain scheduling for cases of unknown scheduling parameters. The mass estimator asymptotically tracks the unknown mass, and the gains of the stabilizing controller are appropriately scheduled by the estimated mass. Thus, this paper demonstrates the capability of
such a control system to achieve both robustness against mass uncertainty, and a performance improvement by suppressing the high-frequency resonance over the conventional control system. In addition, the developed control system can be implemented in magnetically levitated transport systems whose parametric uncertainties are subject to actuator failure of some levitated magnets, high propulsion speed, continuity and flatness of guideway, etc.

Although the track is normally considered fixed in the results, its dynamics needs to be reflected in the design of a reliable control system for moving vehicles. The issue of the track inputs and the associated measurements remain as future works.

Fig. 9. Tracking control for the desired trajectory $r_d = 10 \text{ mm} + 2.5 \sin(2\pi t) \text{ mm}$ without any additional mass. (a) Using conventional state feedback controller. (b) Using self-tuning controller.

Fig. 10. Tracking control for the desired trajectory $r_d = 10 \text{ mm} + 2.5 \sin(2\pi t) \text{ mm}$ with an additional mass of 300 kg. (a) Using conventional state feedback controller. (b) Using self-tuning controller.

References


